Math 323 - Formal Mathematical Reasoning and Writing

Problem Session

Wednesday, 3/4/15

1. Each of the following items asks you to give either a definition or a statement of a theorem. Please give **mathematically precise** statements.
   (a) State the *Archimedean Principle*.
   (b) Let S be a subset of the real numbers. State what it means for a number a to be the *greatest lower bound* of S.
   (c) State the *Triangle Inequality*.
2. Let $A = \{x \in \mathbb{Q} : x < 10\}$. Decide whether the following statements are true or false. Briefly explain your answers.
   (a) If y is not an upper bound of S, then y is in S.
   (b) If y is not an upper bound of S, then y is not in S.
   (c) For every $y \in S$, there exists some $z \in S$ such that $z > y$.
3. Consider the set $C = \left\{\frac{n}{2n+1} : n \in \mathbb{N}\right\}$. Determine, with proof, the least upper bound and greatest lower bound of S.

If you're finished, try this **bonus problem!**

✈ Prove that if $a, b, c \in \mathbb{Q}$ and $a < b$, then $a + c < b + c$.

This is one of the properties of an ordered field. Here, you are trying to prove this property specifically for the field $\mathbb{Q}$. Instead of invoking the fact that $\mathbb{Q}$ is an ordered field, try to prove this using only the definitions of operations on $\mathbb{Q}$.

When you go home, do the following:

- **Step 1** Look through your class notes and your homework, and make a list of all the properties of $\mathbb{Q}$ that you proved. For instance:
  - Addition is commutative, multiplication is associative, every $x \in \mathbb{Q}$ has an additive inverse, etc...
- **Step 2** Now, compare this list to the list of properties in section 4.3.2 of your textbook (pp. 60-63). Make a list of all the properties you have not yet proved in class or on the homework. For instance:
  - Addition is associative, multiplication is commutative, every $x \in \mathbb{Q}$ has a multiplicative inverse, etc...
- **Step 3** Prove all the properties you listed in Step 2.

LaTeX tip of the week!

Are you using LaTeX on Windows and that file I sent you still isn't compiling to give you a pdf? There is a `setup.exe` file that you needed to run. Perhaps your forgot to do that?

Sometimes you want big versions of $(), \{\}$. To do this, use `\left` and `\right`. For instance:

```
\[ \{ \frac1n : n \in \mathbb{N} \} \]
\[ (\frac12)(\frac1n) \]
```

$$\{\frac{1}{n} : n \in \mathbb{N}\}$$

$$(\frac{1}{2})(\frac{1}{n})$$

VS

```
\[ \left\{ \frac1n : n \in \mathbb{N} \right\} \]
\[ \left( \frac12 \right) \left( \frac1n \right) \]
```

$$\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$$

$$\left(\frac{1}{2}\right)\left(\frac{1}{n}\right)$$